Chapter 15
Infinite Impulse Response (IIR) Filters

Learning Objectives

- **Introduction to the theory behind IIR filters:**
  - **Properties.**
  - **Coefficient calculation.**
  - **Structure selection.**
- **Implementation in Matlab, C and linear assembly.**

Introduction

- Infinite Impulse Response (IIR) filters are the first choice when:
  - Speed is paramount.
  - Phase non-linearity is acceptable.
- IIR filters are computationally more efficient than FIR filters as they require fewer coefficients due to the fact that they use feedback or poles.
- However feedback can result in the filter becoming unstable if the coefficients deviate from their true values.

Properties of an IIR Filter

- The general equation of an IIR filter can be expressed as follows:

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_N z^{-N}}{1 + a_1 z^{-1} + \ldots + a_M z^{-M}} = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1 + \sum_{k=1}^{M} a_k z^{-k}}$$

- a_k and b_k are the filter coefficients.

Properties of an IIR Filter

- The transfer function can be factorised to give:

$$H(z) = k\frac{(z-z_1)(z-z_2)\cdots(z-z_N)}{(z-p_1)(z-p_2)\cdots(z-p_N)} = \frac{Y(z)}{X(z)}$$

- Where: $z_1, z_2, \ldots, z_N$ are the zeros,
  $p_1, p_2, \ldots, p_N$ are the poles.

Properties of an IIR Filter

- **The transfer function can be factorised to give:**

$$H(z) = k\frac{(z-z_1)(z-z_2)\cdots(z-z_N)}{(z-p_1)(z-p_2)\cdots(z-p_N)} = \frac{Y(z)}{X(z)}$$

- **For the implementation of the above equation we need the difference equation:**

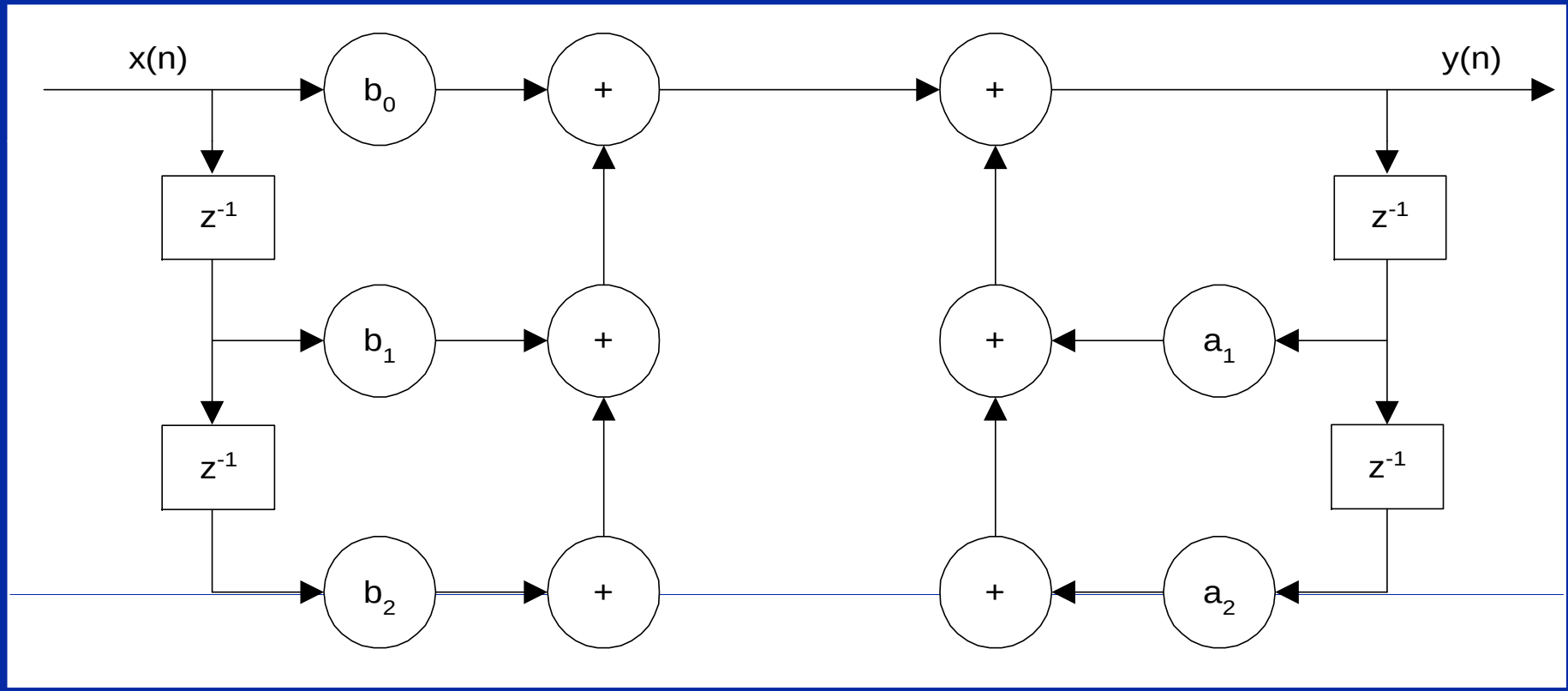
$$y[n] = \sum_{k=0}^{\infty} h[k]x[n-k]$$

$$= \sum_{k=0}^{N} b[k]x[n-k] + \sum_{k=1}^{M} a[k]y[n-k]$$

Properties of an IIR Filter

$$y[n] = \sum_{k=0}^{N} b[k]x[n-k] + \sum_{k=1}^{M} a[k]y[n-k]$$

IIR Equation

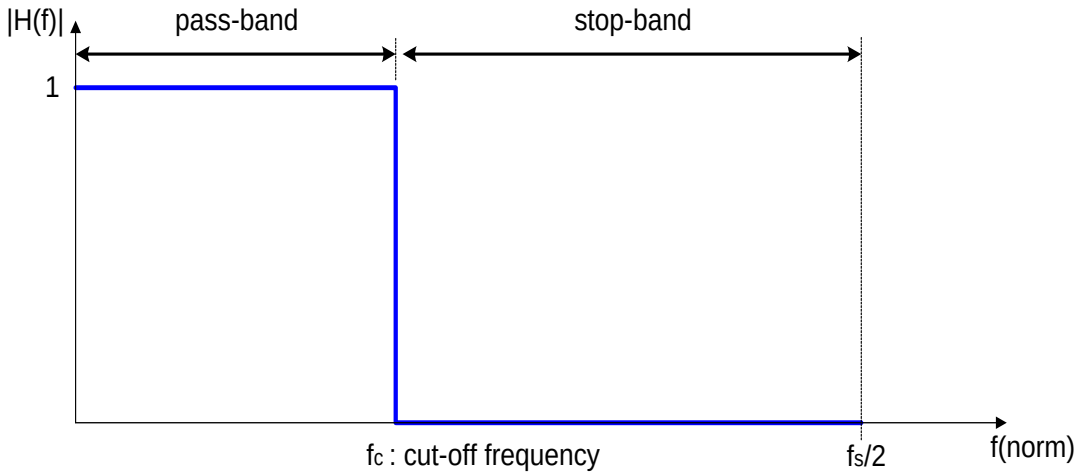


IIR structure for N = M = 2

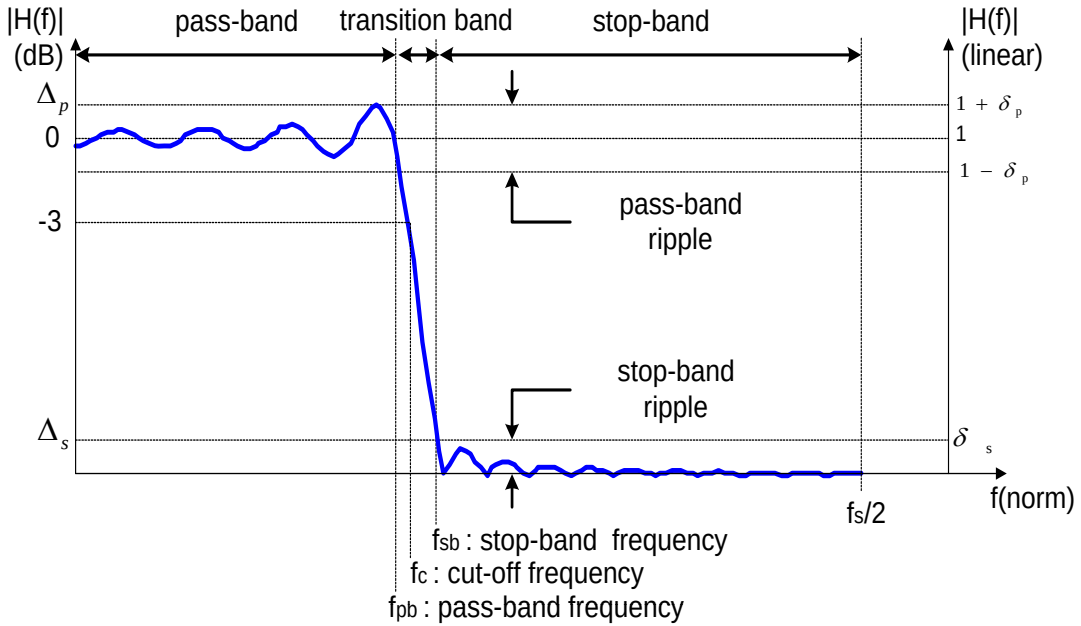
Design Procedure

- **To fully design and implement a filter five steps are required:**
  - **(1) Filter specification.**
  - **(2) Coefficient calculation.**
  - **(3) Structure selection.**
  - **(4) Simulation (optional).**
  - **(5) Implementation.**

Filter Specification - Step 1



(a)



(b)

Coefficient Calculation - Step 2

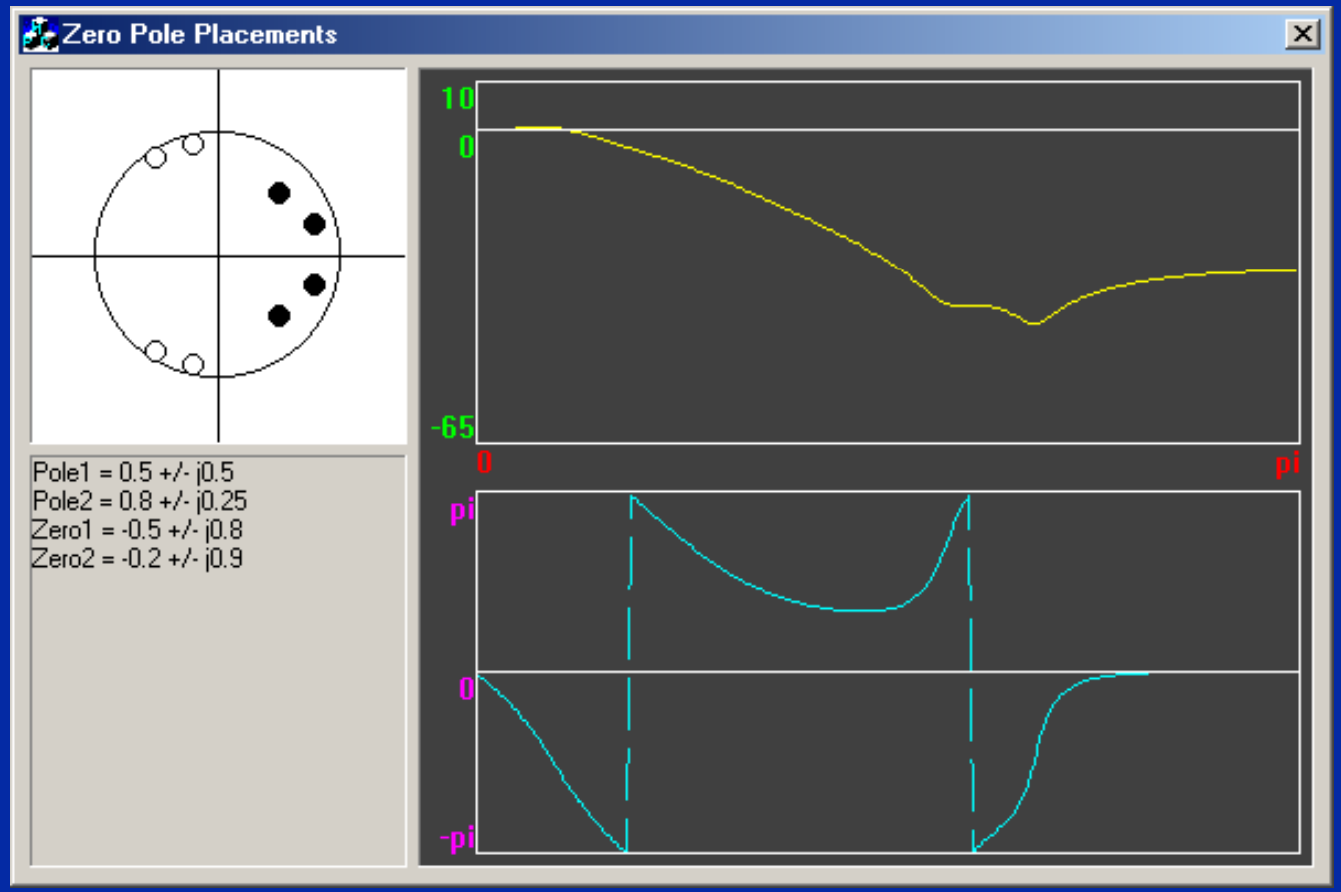
- There are two different methods available for calculating the coefficients:
  - Direct placement of poles and zeros.
  - Using analogue filter design.
- Both of these methods are described.

Placement Method

- All that is required for this method is the knowledge that:
  - Placing a zero near or on the unit circle in the z-plane will minimise the transfer function at this point.
  - Placing a pole near or on the unit circle in the z-plane will maximise the transfer function at this point.
  - To obtain real coefficients the poles and zeros must either be real or occur in complex conjugate pairs.

Placement Method

- **Example - Placement method:**



- **Link: \Links\zeropole.exe**

Analogue to Digital Filter Conversion

- This is one of the simplest method.
- There is a rich collection of prototype analogue filters with well-established analysis methods.
- The method involves designing an analogue filter and then transforming it to a digital filter.
- The two principle methods are:
  - Bilinear transform method (\Links\Bilinear Theory.pdf).
  - Impulse invariant method.

Bilinear Transform Method

- **Practical example of the bilinear transform method:**
  - **The design of a digital filter to approximate a second order low-pass analogue filter is required.**
  - **The transfer function that describes the analogue filter is:**

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

  - **The digital filter is required to have:**
    - **Cut-off frequency of 6kHz.**
    - **Sampling frequency of 20kHz.**

Bilinear Transform Method

- **Matlab code for calculating coefficients:**

```
a   = tan(pi*2/8)           % cut-off 2kHz, fsample 8 kHz, input < 580 mVpp
b   = (1 + 2^0.5 + (a*a))
b00 = (a*a)/b
b01 = 2*b00
b02 = b00
a01 = 2*(a^2 -1)/b
a02 = (1 + a^2 - (2^0.5)*a)/b

bb  = [b00 b01 b02];
aa  = [1 a01 a02];

figure(1)
freqz(bb,aa,512,8000)

fid = fopen('IIR_coef_float.txt', 'w');
    fprintf(fid,'%0.4f,%0.4f,%0.4f\n',bb);
    fprintf(fid,'%0.4f,%0.4f\n',aa);
fclose(fid);
```

Bilinear Transform Method

- **Output from Matlab code:**
  - **bb = [0.1311, 0.2622, 0.1311]**
  - **aa = [1, -0.7478, 0.2722]**
- **Converting these to Q15 format we get:**
  - **$b = (bb * 2^{15})_{HEX}$ = [0x10C7, 0x218F, 0x10C7]**
  - **$a = (bb * 2^{15})_{HEX}$ = [0x7FFF, 0xA049, 0x22D7]**
- **Note that $1 \sim (0x7FFF)_{DEC}$**

Realisation Structures - Step 3

- **Direct Form I:**

$$H(z)=\frac{Y(z)}{X(z)}=\frac{\sum_{k=0}^{N} b_k z^{-k}}{1+\sum_{k=1}^{M} a_k z^{-k}}=\frac{b_0+b_1 z^{-1}+\ldots+b_N z^{-N}}{1+a_1 z^{-1}+\ldots+a_M z^{-M}}$$

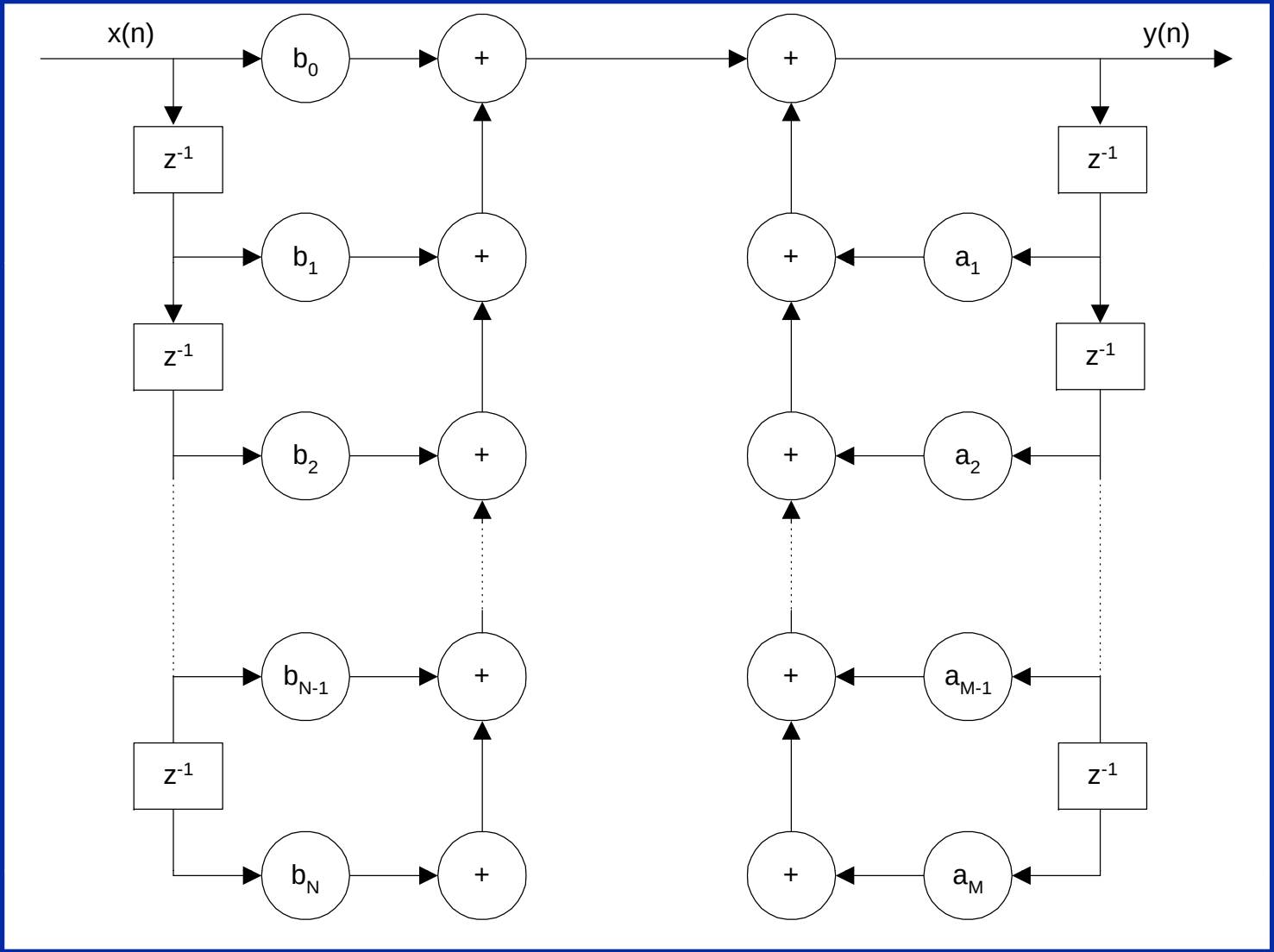
- **Difference equation:**

$$y[n]=\sum_{k=0}^{N} b[k]x[n-k]+\sum_{k=1}^{M} a[k]y[n-k]$$

- **This leads to the following structure…**

Realisation Structures - Step 3

- **Direct Form I:**



Realisation Structures - Step 3

- **Direct Form II canonic realisation:**

$$H(z) = H_1(z)H_2(z) = \frac{1}{1+\sum_{k=1}^{M} a_k z^{-k}} \sum_{k=0}^{N} b_k z^{-k}; \quad \text{for N = M}$$

$$= \frac{P(z)}{X(z)} \cdot \frac{Y(z)}{P(z)}$$

- **Where:**

$$\frac{P(z)}{X(z)} = \frac{1}{1+\sum_{k=1}^{M} a_k z^{-k}} \quad \text{and} \quad \frac{Y(z)}{P(z)} = \sum_{k=0}^{N} b_k z^{-k}$$

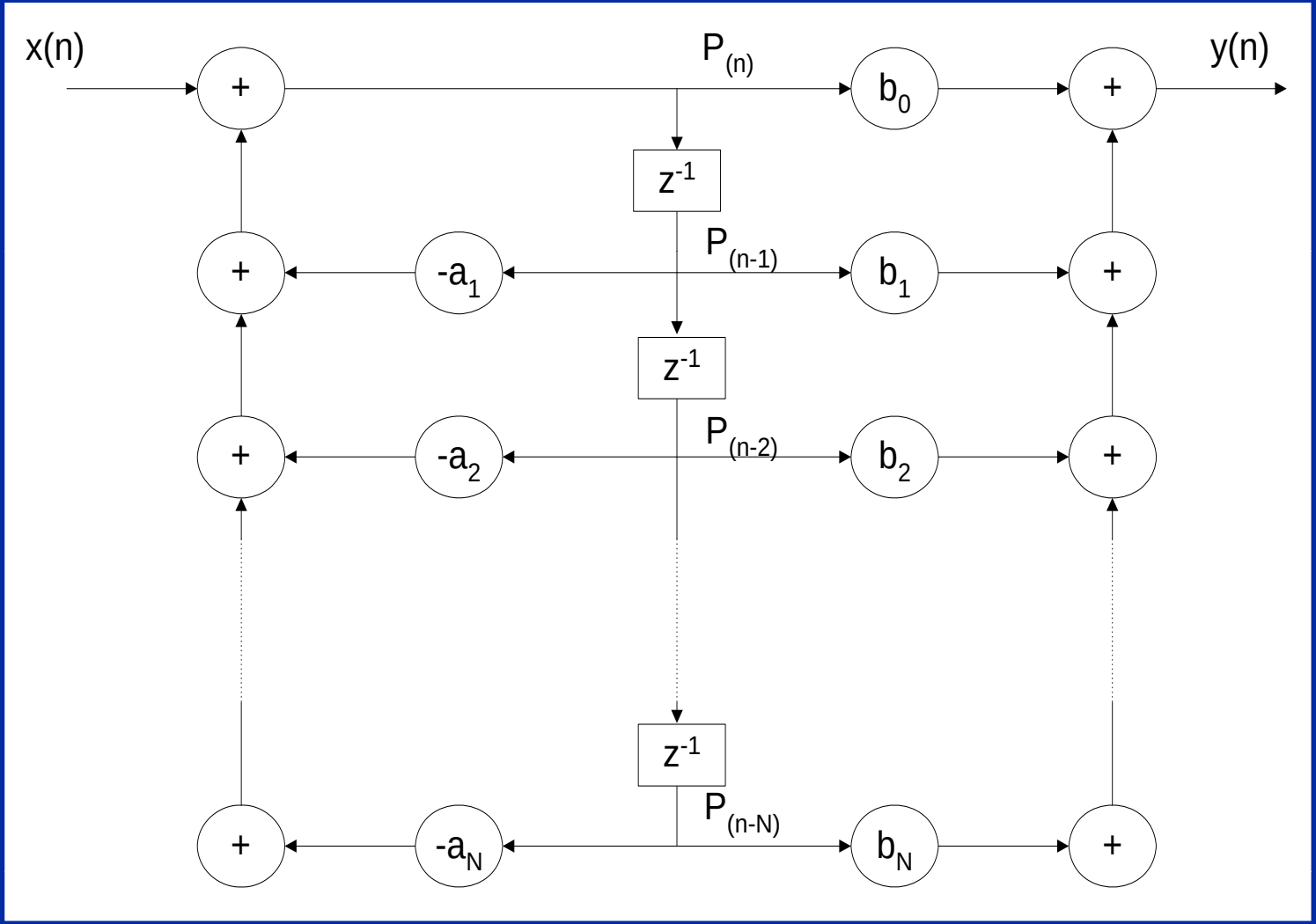
- **Taking the inverse of the z-transform of P(z) and Y(z) leads to:**

$$p(n) = x(n) - \sum_{k=1}^{N} a_k p(n-k)$$

$$y(n) = \sum_{k=0}^{N} b_k p(n-k)$$

Realisation Structures - Step 3

- **Direct Form II canonic realisation:**



Implementation - Step 5

‘C’ code

```
void IIR_Isr (void)
{
      short a1 = 0x0;
      short a2 = 0x15f6;
      short b0 = 0x257d;
      short b1 = 0x4afd;
      short b2  = 0x257d;
      static short d01=0, d02=0, d00;
      short xn, y0;
      int prod1, prod2, prod3, prod4, prod5, input, output;

      input = mcbsp0_read();     // Read the input sample from the serial port
      y0 = 0;
      input &= 0xffff;

      xn = (short) (input & 0x000ffff);
      prod1 = _mpy(d02,a2)>>15;
      prod2 = _mpy(d01,a1)>>15;
      d00 = xn + (short)(prod1 + prod2);
      prod3 = _mpy(d01,b1);
      prod4 = _mpy(d02,b2);
      prod5 = _mpy(d00,b0);
      y0 = (short)((prod3+prod4+prod5)>>15);
      d02 = d01;
      d01 = d00;
      output = y0;

      mcbsp0_write(output& 0xfffffffe);    // Write the signal to the serial port
      return;
}
```

Implementation - Step 5

```
            .def    _iir_sa
            .sect   "mycode"

_iir_sa     .cproc  an1, an2, bn0, bn1, bn2, delays, x_ptr, y_ptr,mask, mask2

            .reg    p0, p1, p2
            .reg    prod1, prod2, prod3, prod4, prod5
            .reg    sum1, sum2, sum3
            .reg    x, ref, y0,y1

            LDW     *x_ptr, x
            AND     x,mask,x
            LDH     *+delays[0], p1
            LDH     *+delays[1], p2
            MPY     an1, p1, prod1
            MPY     an2, p2, prod2
            ADD     prod1, prod2, sum1
            SHR     sum1, 15, sum1
            ADD     x, sum1, p0
            MPY     bn0, p0, prod3
            MPY     bn1, p1, prod4
            MPY     bn2, p2, prod5
            ADD     prod4, prod5, sum2
            ADD     prod3, sum2, sum3
            SHRU    sum3, 15, y0

            STH     p1, *+delays[1]
            STH     p0, *+delays[0]

            AND     y0, mask2, y0
            STW     y0, *y_ptr

            .return y0
            .endproc
```

Linear assembly code

IIR Code

- **Code location:**
  - **Code\Chapter 15 - Infinite Impulse Response Filters**
- **Projects:**
  - **Fixed Point in C: \FIR_C_Fixed**
  - **Fixed Point in Linear Asm: \FIR_Sa_Fixed**

Chapter 15
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- End -